



Pre-Algebra Workbook Solutions

Radicals

RADICALS

- 1. Complete the statement.

Radicals are the opposite of _____.

Solution:

exponents

- 2. $\sqrt{x}$ can also be written as _____.

Solution:

$$x^{\frac{1}{2}}$$

- 3. $\sqrt[4]{x}$ can also be written as _____.

Solution:

$$x^{\frac{1}{4}}$$



- 4. Find the value of $\sqrt{16}$.

Solution:

Rewrite the value under the root as a perfect square.

$$\sqrt{4 \cdot 4}$$

So the square root is

$$4$$

- 5. Find the value of $\sqrt{36}$.

Solution:

Rewrite the value under the root as a perfect square.

$$\sqrt{6 \cdot 6}$$

So the square root is

$$6$$

- 6. Complete the statement.



$x^{\frac{1}{3}}$ can also be written as _____.

Solution:

$$\sqrt[3]{x}$$

■ 7. Find the value of $\sqrt{300}$.

Solution:

Rewrite the value under the root as a product.

$$\sqrt{100 \cdot 3}$$

Now separate the values under the radical into separate radicals.

$$\sqrt{100}\sqrt{3}$$

Rewrite the value under the root as a perfect square.

$$\sqrt{10 \cdot 10}\sqrt{3}$$

So the square root is

$$10\sqrt{3}$$



■ 8. Find the value of $\sqrt{5,000}$.

Solution:

Rewrite the value under the root as a product.

$$\sqrt{100 \cdot 50}$$

$$\sqrt{100 \cdot 25 \cdot 2}$$

Now separate the values under the radical into separate radicals.

$$\sqrt{100}\sqrt{25}\sqrt{2}$$

Rewrite the value under the root as a perfect square.

$$\sqrt{10 \cdot 10}\sqrt{5 \cdot 5}\sqrt{2}$$

So the square root is

$$10(5)\sqrt{2}$$

$$50\sqrt{2}$$



ADDING AND SUBTRACTING RADICALS

- 1. Find the value of $2\sqrt{3} + 5\sqrt{3}$.

Solution:

Because the radicals are the same, we have like-terms and we can add them directly.

$$(2 + 5)\sqrt{3}$$

$$7\sqrt{3}$$

- 2. Find the value of $\sqrt{32} - \sqrt{2}$.

Solution:

Because the radicals aren't the same, we need to simplify the radicals first.

$$\sqrt{16 \cdot 2} - \sqrt{2}$$

$$\sqrt{16}\sqrt{2} - \sqrt{2}$$

$$4\sqrt{2} - \sqrt{2}$$

Now with like terms, we can find the difference.



$$(4 - 1)\sqrt{2}$$

$$3\sqrt{2}$$

- 3. Find the value of $\sqrt{3} + \sqrt{12}$.

Solution:

Because the radicals aren't the same, we need to simplify the radicals first.

$$\sqrt{3} + \sqrt{4 \cdot 3}$$

$$\sqrt{3} + \sqrt{4}\sqrt{3}$$

$$\sqrt{3} + 2\sqrt{3}$$

Now with like terms, we can find the sum.

$$(1 + 2)\sqrt{3}$$

$$3\sqrt{3}$$

- 4. Find the value of $\sqrt{16} + \sqrt{25}$.

Solution:



Both radicals can be evaluated directly.

$$\sqrt{16} + \sqrt{25}$$

$$4 + 5$$

$$9$$

■ 5. Find the value of $4\sqrt{3} + 2\sqrt{2} - 2\sqrt{3} - \sqrt{2}$.

Solution:

If we re-order the terms, we notice that we can combine some of the radicals.

$$(4\sqrt{3} - 2\sqrt{3}) + (2\sqrt{2} - \sqrt{2})$$

$$(4 - 2)\sqrt{3} + (2 - 1)\sqrt{2}$$

$$2\sqrt{3} + \sqrt{2}$$

The remaining roots are still different, and can't be simplified further, so this is as far as we can simplify the expression.

■ 6. Find the value of $3\sqrt{4} - 2\sqrt{9}$.



Solution:

Both radicals can be evaluated directly.

$$3(2) - 2(3)$$

$$6 - 6$$

$$0$$



MULTIPLYING RADICALS

- 1. Find the value of $\sqrt{20} \cdot \sqrt{4}$.

Solution:

Simplify the radicals first,

$$\sqrt{4 \cdot 5} \cdot \sqrt{4}$$

$$\sqrt{4}\sqrt{5} \cdot \sqrt{4}$$

$$2\sqrt{5} \cdot 2$$

then multiply.

$$4\sqrt{5}$$

- 2. Find the value of $\sqrt{13} \cdot \sqrt{7}$.

Solution:

Neither radical can be simplified initially, so simply multiply the radicals.

$$\sqrt{13 \cdot 7}$$



$$\sqrt{91}$$

The value 91 has no factors that are perfect squares, so we can't simplify it any further.

■ 3. Find the value of $8\sqrt{3} \cdot \sqrt{12}$.

Solution:

Multiply the radicals first,

$$8\sqrt{3 \cdot 12}$$

$$8\sqrt{36}$$

then simplify.

$$8(6)$$

$$48$$

■ 4. Find the value of $15\sqrt{2} \cdot \sqrt{16}$.

Solution:

Simplify the radicals first,



$$15\sqrt{2} \cdot 4$$

then multiply.

$$60\sqrt{2}$$



DIVIDING RADICALS

- 1. Simplify the expression.

$$\sqrt{\frac{36}{6}}$$

Solution:

Simplify the quotient inside the radical.

$$\sqrt{6}$$

There are no factors of 6 that are perfect squares, so we can't simplify the radical any further.

- 2. Simplify the expression.

$$\sqrt{\frac{45}{5}}$$

Solution:

Simplify the quotient inside the radical, then simplify the radical.

$$\sqrt{9}$$



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■ 3. Simplify the expression.

$$\frac{1 + \sqrt{5}}{\sqrt{5}}$$

Solution:

Break up the fraction into separate fractions.

$$\frac{1}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{5}}$$

$$\frac{1}{\sqrt{5}} + 1$$

We never want to leave a radical in the denominator if we can avoid it, so we'll multiply both the numerator and denominator of the first fraction by $\sqrt{5}$. Multiplying the numerator and denominator by the same value is like multiplying by 1, so we won't actually be changing the value of the fraction, just rewriting it in a different form.

$$\frac{1}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right) + 1$$



$$\frac{1\sqrt{5}}{\sqrt{5}\sqrt{5}} + 1$$

$$\frac{\sqrt{5}}{5} + 1$$

- 4. Simplify the expression.

$$\frac{\sqrt{25}}{\sqrt{5}}$$

Solution:

The quotient of roots is always equal to the root of the quotient, so we can rewrite the expression as

$$\sqrt{\frac{25}{5}}$$

Simplify the quotient.

$$\sqrt{5}$$



RADICAL EXPRESSIONS

- 1. Find the value of $\sqrt{45}$.

Solution:

Look for perfect square factors of 45, and rewrite the value inside the radical.

$$\sqrt{9 \cdot 5}$$

Split the values into separate radicals, then simplify.

$$\sqrt{9}\sqrt{5}$$

$$3\sqrt{5}$$

- 2. Find the value of $\sqrt{80} - \sqrt{20}$.

Solution:

Notice that $80 = 4 \cdot 20$, and use that to simplify the first radical.

$$\sqrt{4 \cdot 20} - \sqrt{20}$$

$$\sqrt{4}\sqrt{20} - \sqrt{20}$$



$$2\sqrt{20} - \sqrt{20}$$

Now that the radicals are the same, combine like terms.

$$(2 - 1)\sqrt{20}$$

$$1\sqrt{20}$$

$$\sqrt{20}$$

Now simplify the remaining radical.

$$\sqrt{4 \cdot 5}$$

$$\sqrt{4}\sqrt{5}$$

$$2\sqrt{5}$$

■ 3. Find the value of $5\sqrt{24} \cdot \sqrt{15}$.

Solution:

First simplify the radicals individually.

$$5\sqrt{4 \cdot 6} \cdot \sqrt{15}$$

$$5\sqrt{4}\sqrt{6} \cdot \sqrt{15}$$

$$5(2)\sqrt{6} \cdot \sqrt{15}$$



$$10\sqrt{6} \cdot \sqrt{15}$$

Now factor the values inside both radicals, then split the radicals apart, group together like terms, and simplify.

$$10\sqrt{2 \cdot 3} \cdot \sqrt{3 \cdot 5}$$

$$10\sqrt{2}\sqrt{3} \cdot \sqrt{3}\sqrt{5}$$

$$10(\sqrt{3}\sqrt{3})\sqrt{2}\sqrt{5}$$

$$10(3)\sqrt{2}\sqrt{5}$$

$$30\sqrt{10}$$

■ 4. Complete the statement.

$\sqrt{xy}$ can also be written as _____.

Solution:

The root of a product can be rewritten as the product of the roots, so we can rewrite the expression as $\sqrt{x}\sqrt{y}$.

■ 5. Complete the statement.



$\sqrt{\frac{m}{n}}$ can also be written as _____.

Solution:

The root of a quotient can be rewritten as the quotient of roots, so we can rewrite the expression as

$$\frac{\sqrt{m}}{\sqrt{n}}$$

■ 6. Complete the statement.

The square root of a number multiplied by the square root of the same number is equal to _____.

Solution:

The square root of a number multiplied by the square root of the same number is equal to the number itself. For example,

$$\sqrt{5}\sqrt{5} = 5$$

■ 7. Find the value of $\sqrt{2} + \sqrt{32} - \sqrt{50}$.



Solution:

First simplify the radicals individually.

$$\sqrt{2} + \sqrt{16 \cdot 2} - \sqrt{25 \cdot 2}$$

$$\sqrt{2} + \sqrt{16}\sqrt{2} - \sqrt{25}\sqrt{2}$$

$$\sqrt{2} + 4\sqrt{2} - 5\sqrt{2}$$

Now that all the radicals are the same, combine like terms.

$$(1 + 4 - 5)\sqrt{2}$$

$$(0)\sqrt{2}$$

$$0$$

■ 8. Complete the statement.

To be able to add or subtract radicals, the roots must be _____ when they are simplified.

Solution:

To be able to add or subtract radicals, the roots must be identical when they are simplified. In other words, we can could simplify an expression like



$$4\sqrt{2} + 3\sqrt{2}$$

because the radicals are identical, but we couldn't simplify

$$3\sqrt{2} - 4\sqrt{5}$$

because the radicals are different and they can't be rewritten in a way that would make them identical.

■ 9. Roberta is trying to simplify the following radical expression,

$$\sqrt{4} + \sqrt{20} - 2\sqrt{5} + \sqrt{25}$$

and her work is shown below.

$$\text{Step 1: } 2 + \sqrt{20} - 2\sqrt{5} + 5$$

$$\text{Step 2: } 2 + \sqrt{4 \cdot 5} - 2\sqrt{5} + 5$$

$$\text{Step 3: } 2 + 4\sqrt{5} - 2\sqrt{5} + 5$$

$$\text{Step 4: } 7 + 2\sqrt{5}$$

In which step did she make a mistake? What should she have done differently, and what is the correct answer?

Solution:

Roberta made her mistake in Step 3. Instead of writing $4\sqrt{5}$, she should have written $2\sqrt{5}$, because she should have simplified $\sqrt{4}$ to 2, instead of



simplifying it to 4. Starting from Step 2, her work should have looked like this:

$$2 + \sqrt{4 \cdot 5} - 2\sqrt{5} + 5$$

$$2 + \sqrt{4}\sqrt{5} - 2\sqrt{5} + 5$$

$$2 + 2\sqrt{5} - 2\sqrt{5} + 5$$

$$2 + 5$$

$$7$$



